Effective usage of credit records promotes cooperation on weighted networks

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The cooperative behaviors of players on weighted networks are investigated by incorporation of trust mechanisms into a well-accepted game model, i.e., the networked prisoner's dilemma game, afterwards some weight-updating schemes are designed according to the credit records. Despite the differences in network topologies and strategy updating protocols, a simple yet significant principle surfaces that, to promote the emergence of cooperation over abundant weighted networks, only the latest credit record of partners is required to be taken into consideration, whereas incorporating more previous records may even deteriorate the cooperation performance. To support such an appealing principle, we have investigated more deeply into the role of credit records so as to give a detailed explanation underlying it. The virtue of this work lies in providing insights into the effective usage of the currently available credit records.

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I. INTRODUCTION

It is of great challenge and interest for biologists, social scientist, and physicists to investigate the ubiquitous phenomenon that selfish individuals would cooperate with others despite of the high cost from their defection in both biological swarms or flocks and human societies. Consequently, in recent years, many scholars in relevant fields have devoted themselves to evolutionary game theory aiming at providing a powerful mathematical framework to address the emergence of cooperation therein [1–3].

Among various gaming theory models, the evolutionary prisoner's dilemma game (PDG) is most frequently used [4–7]. It is well known that natural selection means a fierce competition between individuals, and hence cooperation hardly emerges, unless some internal mechanisms play a role to facilitate the collaboration. Actually, four internal mechanisms of the cooperation evolution, i.e., direct reciprocity, indirect reciprocity, kin selection, and group selection, have been extensively investigated with good application potential in real lives [8]. Additionally, by introducing evolutionary game theory into complex networks or reverse, scholars found spatial structures or network reciprocities have great influence on the global performance of players on different kinds of networks ranging from regular lattice and random graph to scale-free networks [9–18]. Furthermore, a simple rule is proposed by [8] that determines whether network reciprocity facilitates cooperation or not. On the other hand, due to the characteristics of growth and preferential attachment in dynamic network reciprocity, it is demonstrated by [12,13] that cooperation always dominates defection over scale-free networks.

Promisingly, recent studies on the effect of heterogeneous property endow us a deeper insight into the cooperation behavior over scale-free networks [5,7,13–16]. For instance, by introducing suitable amounts of conformity into the strategy updating rule, cooperators will no longer be dominant, because the less biased information flow makes hubs more susceptible to less-connected partners [5]. As a continual work, the dynamical organization of cooperation on scale-free networks has been investigated in [13] and it is found that players can be partitioned into three subsets: pure cooperators, pure defectors, and those that frequently change strategies. Surprisingly, pure cooperators always form a single cluster, which includes the most connected individuals. On the other hand, extensive efforts were devoted to the coevolution of strategies driven by imitation rules and network topologies [19–25]. Zimmermann *et al.* [19] introduced a model, in which interactions between players are modeled by PDG, and the network topology evolves adaptively according to the payoff of each player. With such protocol, it is observed that the network structure, which eventually becomes hierarchical, benefits from the evolution of cooperation, whereas it is vulnerable to specific attacks. Actually, it has been discovered recently that heterogeneities among players, which are generated spontaneously via evolving interaction networks, are important promoters of cooperation [26,28–30,33–35]. Moreover, in recent years some simple evolutionary rules [26] have also been explored including dynamical interactions [27–30], population growth [31], mobility and aging of players [32–34], and evolving teaching activity [35].

Still worth-mentioning are some other works addressing the essential role of evolutionary mechanisms such as preferential selection [36], memory [37], and teaching ability [38–40] on the evolution of cooperation. Wang *et al.* [37] proposed a memory-based snowdrift game, in which each player records the optimal strategy into its memory by self-

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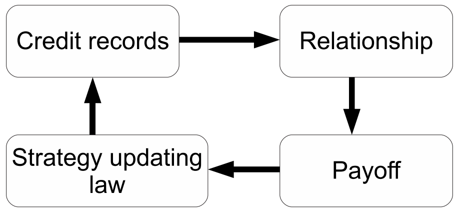

FIG. 1. Interactions of the four ingredients among the present game model. In detail, relationship situation determines the corresponding payoffs, while the payoffs influences the strategy updating law in the next step. The strategy each player adopts will eventually be stored in his or her credit records.

questioning in every round, and the probability of choosing cooperation or defection for each player in the next round depends on the ratio of numbers of cooperation and defection in its memory. Interestingly, they found moderately encouraging selfish behaviors can help to promote cooperation. Szabó *et al.* [39,40] investigated the effect of asymmetry in strategy transfer on the level of cooperation.

Generally speaking, it is difficult for individuals to devise a perfect response strategy without any credit information about their partners, thus the trust mechanism plays a crucial role in the cooperation between individuals. For decades, many researchers in the area of computer science, management science, and even cognitive science have endeavored to investigate the influence of trust on relationships among biological and social groups [41–44]. As one of the most inspiring findings, it is discovered that the interpersonal trust can emerge spontaneously and afterwards evolve with a self-organized dynamical behavior, whose coevolution can effectively enhance the level of cooperation [44]. Unfortunately, the quantitative relation between the trust effect and the interpersonal cooperation has not been intensively investigated so far.

Bearing in mind of the above-mentioned complicated and urgent task, this paper will address the following two major problems therein:

(P1) How does trust mechanism influence the relationship among people.

(P2) How to make full use of people's credit record to promote the interpersonal cooperation.

To address P1, we propose a formula in which each individual adjusts relationship with others merely according to his or her partners' credit records. Clearly, the better credit the partner has, the closer relationship he or she is inclined to keep, which is consistent with the observations in social activities. Figure 1 illustrates the framework of the present network gaming model clearly. Obviously, the evolutionary rule in this paper are based on dynamical interactions mentioned above [26] since the relationship between each pair of players is adjusted online according to the their variational credit records. Regarding P2, in order to put our investigation into a more general scenario, we analyze four different types of networks embedded with two frequently used strategy updating rules, whose results reveal a simple yet general principle with promising application potential in social activities. For convenience, we hereby define two terms "credit records" and "trust mechanism" as below, which are frequently used in this paper. Credit records denote the strategies (cooperation or defection) a player adopted in previous game process and are usually stored in the record file. Their references can be found in the behavior records of citizens and corporations kept in the credit bureau. On the other hand, trust mechanism represents a kind of restriction mechanism, in which credit records of players are used to constrain the dishonest behaviors and to foster cooperation.

II. MODEL

Due to its popularity, we hereby adopt PDG as a platform to study interpersonal relationships, of which mutual cooperation or defection will yield reward R or punishment P to both, respectively. If one player defects while the other cooperates, the defector will receive rewards b and the cooperator will get punishment S with

$$b \geq R \geq P \geq S. \tag{1}$$

To facilitate our investigation, we adopt the weak prisoner's dilemma game by setting $P=S=0$, $R=1$, and b according to [1,2,45]. The temptation of defection, i.e., b, is the single adjusting parameter. In order to observe the cooperative behaviors on networks, we allow $b>2$ despite that it is not a proper PDG. To put our analysis into a sufficiently general scenario, we consider four different networks, i.e., two types of square lattices with periodic boundary conditions (the von Neumann lattice with $\langle k \rangle=4$ and the Moore lattice with $\langle k \rangle=8$), a scale-free network with $\langle k \rangle=4$, and a complete graph. Without loss of generality, we set the sizes of the first three networks as $N=10\,000$ and size of the last network as $N=200$.

In this model, each node represents a player having a credit file, where the most recent strategies are preserved, and each edge $\{i,j\}$ owns a weight ω_{ij} determined by the trust of the two players at both ends i and j of it. Here, ω_{ij} is obtained according to the corresponding credit records as

$$\omega_{ij} = D_i \frac{e^{C_j/L}}{\sum_{k \in \mathcal{N}_i} e^{C_k/L}},$$

where i is the identity number of an arbitrary player, D_i and C_i represent its degree and times of cooperation in the credit file, respectively, and L and $\mathcal{N}_i$ denote the length of credit records and the set of neighbors' identity number of player i, respectively. Since ω_{ij} and ω_{ji} are allocated by plays i and j, respectively, it may happen that $\omega_{ij} \neq \omega_{ji}$. Considering the real state of relationship between two players usually depends on the player who thinks more negatively of their relationship in social life, we hereby set the smaller one as the actual weight $\bar{\omega}_{ij}=\bar{\omega}_{ji}=\min(\omega_{ij},\omega_{ji})$. From the beginning, the weight of each edge is set as 1, which implies that the initial strategy adopted by each player is in cooperation. Note that, in each running step, all the players update their strategies simultaneously, and the credit file only memorizes the recent L strategy records. Thereby, previous strategies in the record file will be replaced by the new ones sequentially, as shown in Fig. 2.

With the above-mentioned preliminary concepts and formulations, we are ready to provide the detailed updating pro-

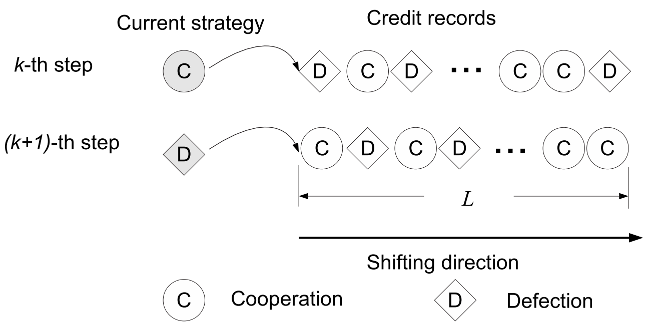

FIG. 2. Updating procedure of the credit records. At the kth running step, assume the current strategy of a player is in cooperation shown by the gray circle. At the $(k+1)$th running step, each record simultaneously shifts backward by one slot, and hence the last record shifts out of the record file. Meanwhile, at the kth running step, the current strategy of the cooperative player shown by the gray circle shifts into the first slot of the file, and at the $(k+1)$th running step, the current strategy of the defective player shown by the gray diamond comes into being. By this means, the updating strategy is established.

tocol of the present model as below. First, the total payoff of player i is calculated by $E_i=\Sigma_{j\in\mathcal{N}_i}B_{ij}\bar{\omega}_{ij}$, with B_{ij} being the payoff of player i by gaming with player j. Before launching its iteration, according to the well-known Fermi rule (FR) [9], player i selects another player j randomly from its neighborhood $\mathcal{N}_i$, and then adopt the strategy of player j with the probability $P_{i\to j}=\frac{1}{1+e^{(E_i-E_j)/K}}$, where K is the external noise magnitude characterizing the irrational choice of the players. To examine the generality of the strategy updating rule's effect on the cooperation performance, we also adopt another famous updating protocol, i.e., proportional imitation rule (PIR) [46,47], in comparison with FR as below

$$P_{i\to j}=\begin{cases}\dfrac{E_j-E_i}{\phi} & \text{if } E_j>E_i\\ 0 & \text{otherwise,}\end{cases} \tag{2}$$

with $\phi=\max(D_i,D_j)b$. So far the present model is detailedly illustrated, and we are now prepared to investigate the influence of credit length on the cooperation behavior over weighted networks.

III. SIMULATIONS AND ANALYSIS

To begin with, in order to evaluate the emergence of cooperation, we introduce another essential parameter, i.e., cooperation frequency ρ_c. Then, we hereby examine four different types of networks, i.e., two square lattices with different average degrees, a scale-free network and a complete graph, by using FR and PIR, respectively. Various schemes are applied on these networks with different lengths L of credit records. As an extreme case, $L=0$ means that the weight of players will not be influenced by the previous credit records and will thereby keep at 1, which is retrogressed into an unweighted network case. Note that all curves here averages over the last 5000 running steps of 20 independent runs of which each has 10 000 running steps,

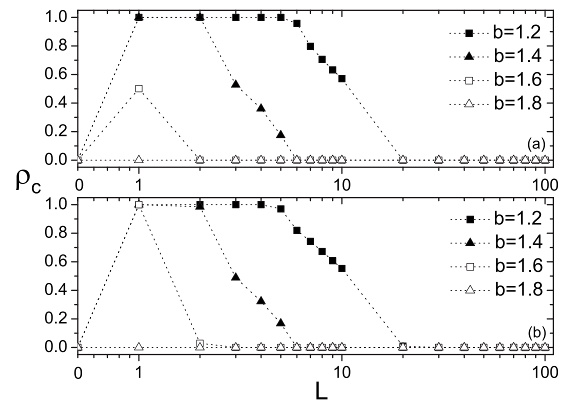

FIG. 3. The frequency of cooperation ρ_c vs L on square lattice with (a) $\langle k\rangle=4$ and (b) $\langle k\rangle=8$. Here, FR is adopted with $K=0.1$, and b represent the temptation of defection with definition given in Eq. (1).

and these 20 runs are implemented at 10 different network realizations with two runs for each realization.

Figures 3(a) and 3(b) show the tendency of cooperation frequency ρ_c with respect to L on square lattices with average degree $\langle k\rangle=4$ and 8, respectively, by adopting FR. Due to the simple architecture of square lattices, we concentrate on the nonlinear dynamics complexity rather than the network topology complexity [48]. Three appealing phenomena can be observed in both figures: (i) If $L=0$, the frequency of cooperation ρ_c is 0 irrelevant to b, which implies that the resisting power to the defection temptation of players adopting such a strategy is quite weak; (ii) along with increasing L, cooperation frequency ρ_c rises from the beginning until reaching a peak and afterwards gradually decreases to zero, showing that the trust mechanism really has a great effect on the emergence of cooperation; (iii) the most attractive observation is that the system has the strongest power to resist the defection temptation when players allocate their corresponding weights merely by examining the latest credit record of their neighbors (corresponds to $L=1$). This is exciting since it reveals a simple yet valuable principle, say, to enhance the cooperation frequency, players do not need too much credit record, and it suffices to consider the latest credit record of their partners. In other words, cooperation can be boosted with low cost of credit memory. Essentially, the promotion of cooperation mainly attributes to the positive feedback between cooperators provided that credit records are taken into account. Let us consider an arbitrary link between two cooperators. If both of them insist on the cooperative strategy, the C records will increase in their credit files, therefore, the weight of the link will enhance and the individuals will obtain more payoffs from their mutual cooperation, which contributes to the adoption of cooperative strategy in the next round. Thereby it will form a positive feedback loop as follows: the more C records exist in individuals' memories, the steadier cooperative relationship is formed between individuals. Particularly, when $L=0$, the frequency of cooperation is naturally low without the aid of the weight assignment rule. Then, for an individual with $L>0$, we consider the influence of the strategy changing from C to D. If only the latest credit record ($L=1$) is considered, the individual can immediately

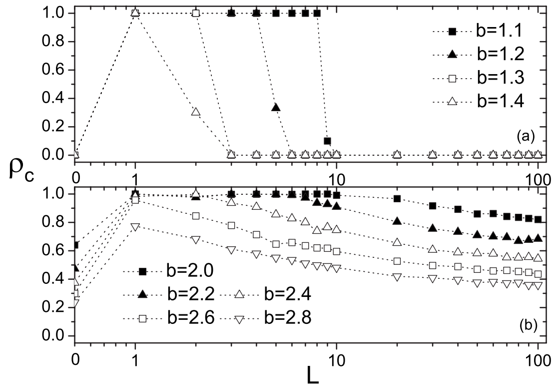

FIG. 4. The frequency of cooperation ρ_c vs the length of credit records L on (a) a complete graph and (b) a scale free network for FR with $K=0.1$.

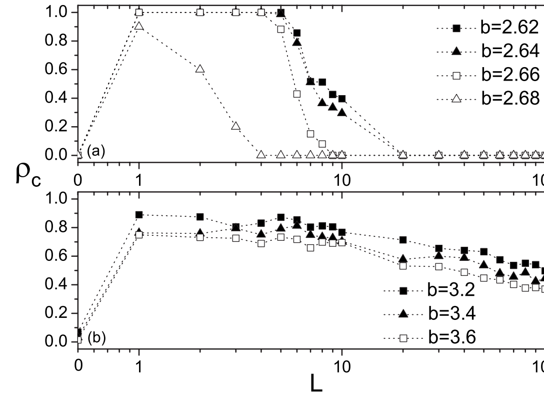

FIG. 6. The frequency of cooperation ρ_c vs L on (a) a complete graph and (b) a scale-free network by adopting PIR.

detect the strategy change in its partner, which leads to a remarkable decrease in the link weight and hence helps to inhibiting the expansion of defection. On the contrary, if L grows larger than 1, the weight on the link will be less sensitive to the strategy change in players due to the previous C records, and hence the cooperation will be undoubtedly weakened. Therefore, the optimal level of cooperation will occur at $L=1$. Interestingly, an inference can be derived that the prompt reaction of individuals to the deteriorated surroundings may contribute to group solidarity.

One may be curious about the situations of more complex networks often encountered in nature or social lives. Bearing in mind of such a concern on the generality of our observation, as shown in Fig. 4, we also display ρ_c along with increasing L on a complete graph and a scale-free network, respectively. It is evident that tendency of ρ_c almost does not change, except for the increased power of resisting the defection temptation compared with the other three network structures due to its special topology [12]. Significantly, the scheme with $L=1$ also yields the best cooperation performance in resisting defection temptations. More promisingly, as shown in Figs. 5 and 6, PIR also produces similar results as FR with a greatly enhanced cooperation level on all these four types of networks, which shows the independence of our observation on the strategy updating law. Therefore, this robust principle represents the "mean-field" behavior of the present model.

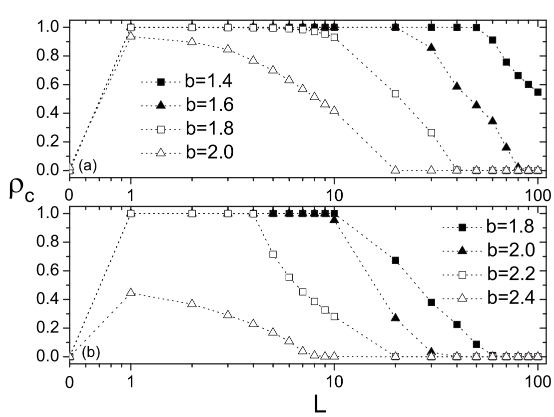

FIG. 5. The frequency of cooperation ρ_c vs L on (a) a square lattice with $\langle k\rangle=4$ and (b) a square lattice with $\langle k\rangle=8$ by adopting PIR.

Hereafter, we will explain the working procedure of the trust mechanism in more detail. As shown above, ρ_c merely depends on two key factors, one is the temptation of defection, namely, b, and the other is the reward-punishment rule, which could be realized by various means [49]. Hereby, we actualize the mechanism by modulating the weights of edges, which would lead to the variations of ρ_c. First, we introduce a concept "effective weight," which is activated only if two cooperators meet or a cooperator meets a defector. For the former case, both players would gain benefits through this edge, and the corresponding weight will thereby become effective for both. Regarding the latter situation, the effective weight would just be allocated to the defector. Obviously, there is no effective weight between two defectors. A typical example is shown in Fig. 7(b), where ω_1, ω_3, and ω_5 are effective weights solely to the defectors, whereas ω_2 and ω_4 are effective weights for the cooperators.

To analyze more deeply into the ability of restraining defection under different strategies, we introduce a parameter to quantify it as $\psi_1=\langle\omega\rangle_D/\langle\omega\rangle_C$, where $\langle\omega\rangle_C$ and $\langle\omega\rangle_D$ represent the average of effective weights of cooperators and defectors, respectively. For example, in Fig. 7(b), $\langle\omega\rangle_C=(\omega_2+\omega_4)/2$ and $\langle\omega\rangle_D=(\omega_1+\omega_3+\omega_5)/3$. For a player bearing comparatively large effective weights, he or she will reap more benefits during the gaming, and hence the player is inclined to keep his or her previous strategy in the following step. In this sense, ψ_1 can be comprehended as defectors'

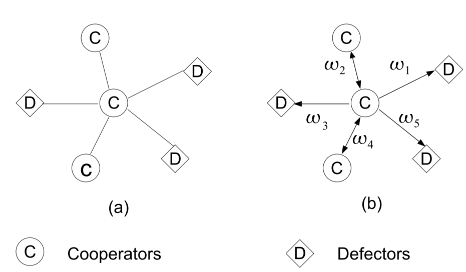

FIG. 7. Illustration of effective weights. (a) The case of unweighted network. (b) The case of weighted network, and the directions of the edges represent the descriptions of effective weights.

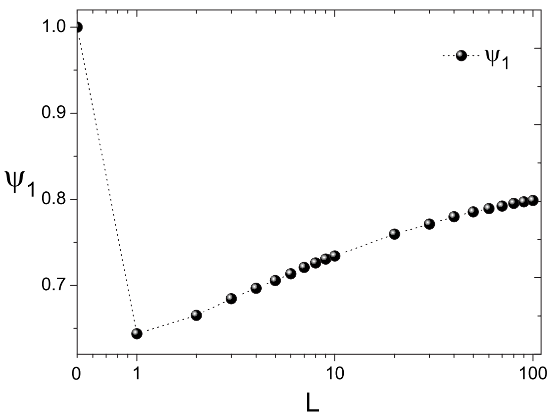

FIG. 8. ψ_1 vs the length of credit records L on scale-free networks for FR with $b=2.8$ and $K=0.1$.

ability to keep the previous strategy with respect to cooperators. According to its definition, the larger ψ_1 is, the more payoffs defectors will be got on average, which implies that players are more likely to prefer defection in face of defection temptations as it is easier for defectors to survive. Therefore, the system has lower frequency of cooperation ρ_c for comparatively large ψ_1. To show more vividly about the ability of restraining defection, we exhibit ψ_1 in Fig. 8 as the function of the credit record length L on a scale-free network, and it is observed that the minimum of ψ_1 is achieved at $L=1$, which is coincide with the peak of ρ_c since defectors get the least average payoffs compared with cooperators in such a situation. Moreover, ψ_1 gradually grows up with increasing L beyond 1, making ρ_c decrease asymptotically. In this way, the main result of the paper is strongly supported that, to optimize the cooperation performance over abundant weighted networks, only the latest credit record of partners is required, and incorporating more previous records or increasing memory cost will however deteriorate the situation.

IV. CONCLUSION AND DISCUSSION

In social activities, trust mechanisms play an essential role in restraining people's defection, and people usually adjust the relationship with their partners according to the corresponding credit records. To investigate more deeply into the role of trust mechanisms, we proposed a weighted network model to vividly describe the dynamic relationship among people. It is demonstrated by extensive numerical simulations that the frequency of cooperation is greatly enhanced by introducing some suitable trust mechanism.

With the assistance of such a network model, an appealing principle is revealed that, with reliable credit records in hand, only the latest credit record is required to achieve the most powerful resistance to temptation of defection. More promisingly, this principle works for three mainstream complex networks including square lattices, Barabási and Albert (BA) scale-free networks and complete graph, with different updating laws including FR and PIR. The generality over various human relationship networks is thus demonstrated. Thereby, this robust principle represents the mean-field behavior of the present game model. To extract the rule behind such an attractive principle, we provide a reasonable explanation by injecting effective weights into the edges of the network models. All these numerical simulation and theoretical analysis lead to a simple yet significant conclusion that the scheme merely considering the latest record has the best performance to promote cooperation, which has nice potential in the social and economic activities relevant to trusting mechanisms. Compared with the tit-for-tat (TFT) in the two-player version of PDG [4], which is based on direct reciprocity, we are interested in network reciprocity and focus on the overall performance of networks (i.e., the frequency of cooperation). Additionally, the defective behavior of partners is suppressed indirectly by dynamically adjusting weights on the links in our model rather than "an eye for an eye" strategy.

This work provides a starting point aimed at effectively promoting interpersonal cooperation by making full use of the available credit records, and we hope that it will open new avenues in this fascinating direction.

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